On the physical DGLAP evolution of structure functions

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in collaboration with M. Stratmann
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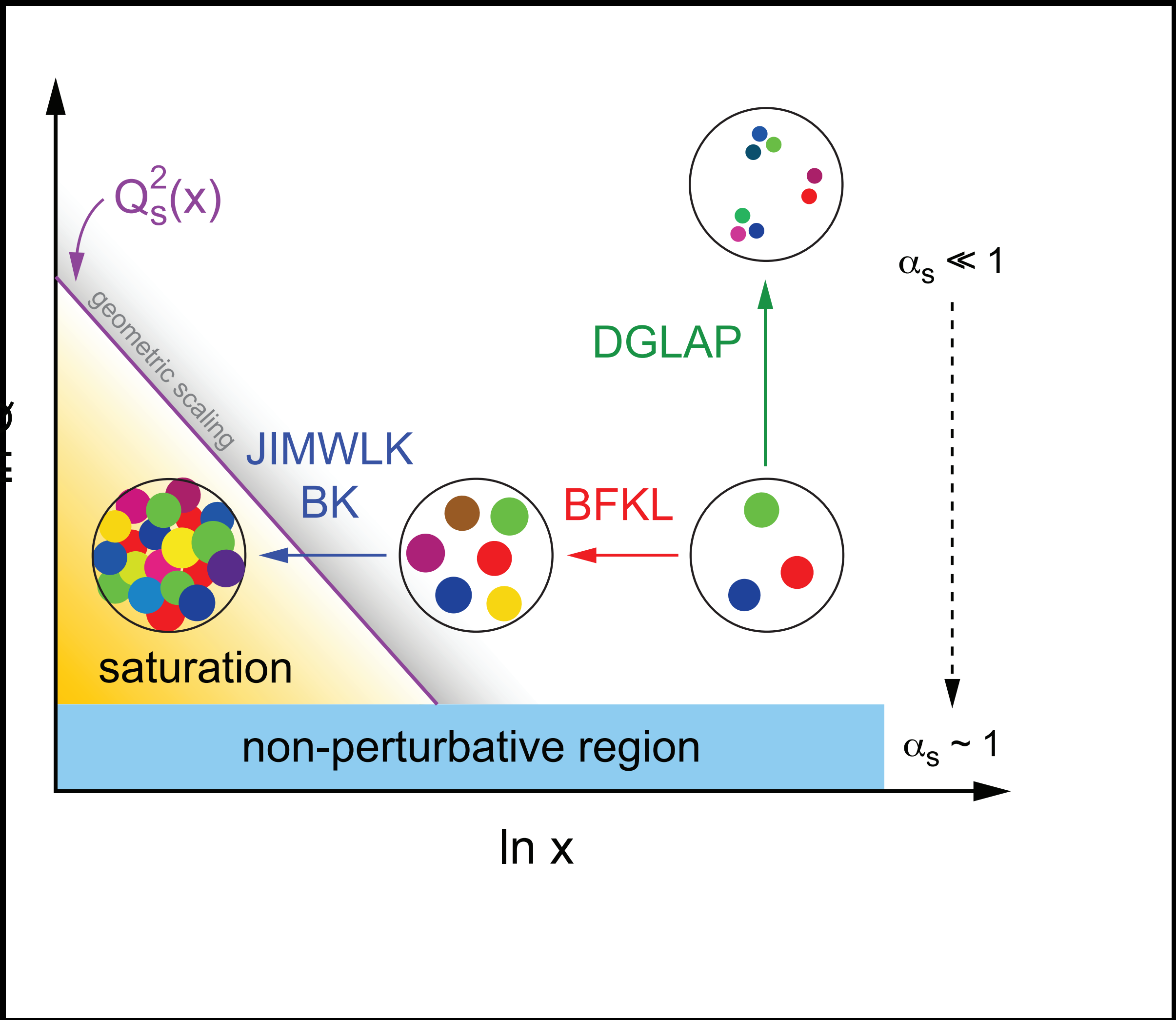
inclusive Deep Inelastic Scattering & evolution equations



Picture:

•DGLAP drives structure function from smaller to larger Q^2

Reality:

• model pdf at initial Q^2
• evolve to higher Q^2
• fit initial conditions until agreement with data

Can we do better?

an old idea: physical DGLAP evolution

idea: don't care about pdfs

↓

evolve observable itself

[Furmanski, Petronzio, ZP C 11, 293(1982)], [Catani, ZP C 75, 665 (1997)], [Blümlein, Ravindran, van Neerven, NPB 586, 349 (2000)]

$$\text{“}Q^2 \frac{d}{dQ^2} F(x, Q^2) = K \otimes F(x, Q^2)\text{”}$$

↑

observable itself!

evolution kernels K

- physical
- no factorization scheme ambiguity; only renormalization scale

→

equivalent to [Catani, ZP C 75, 665 (1997)]

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

from pdf to physical evolution

Physical evolution: write DGLAP equation not in terms of pdfs but observables e.g. structure functions

- Need for every active pdf an observable
- 3 active flavors → 7 observables (more accurate: each flavor combination ≅ an observable)

Reminder: Decoupling of DGLAP through flavor decomposition

DGLAP evolution is matrix valued equation $\longrightarrow$ different quark and gluon pdfs mix

$$\frac{d}{d\ln\mu^2} f_k(x,\mu^2) = \sum_{l=q,g} P_{kl} \otimes f_l(x,\mu^2)$$

Decomposition into quark singlet & various flavor non-singlets

$$\Sigma = \sum_{f=1}^{n_f} (q_f + \bar{q}_f)$$

$$q^{\pm}_{\mathrm{ns},ik} = q_i \pm \bar{q}_i - (q_k \pm \bar{q}_k) \ , \quad q^{\mathrm{v}}_{\mathrm{ns}} = \sum_{r=1}^{n_f} (q_r - \bar{q}_r)$$

$$\frac{d}{d\ln\mu^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

Decoupled evolution for non-singlets & 2-dim evolution for flavor singlet vector (Σ, g)

Our study: the flavor singlet of structure functions

- $(\Sigma, g) \rightarrow$ flavor singlet of 2 structure functions e.g. (F_2^S, F_L^S) or (F_2^S, F_D^S)

- In general: additional measurements (neutron structure functions, F_3, etc.) to subtract non-singlets

- at small x: to good accuracy ≅ full structure functions

Relation pdf ⇔ observable through coefficients

next generation Electron Ion Colliders: (F_2, F_L) with high precision;

➡ use to substitute doublet (Σ, g)

$$\begin{pmatrix} F_2 \\ F_L \end{pmatrix} = \underbrace{\begin{pmatrix} C_{2q} C_{2g} \\ C_{Lq} C_{Lg} \end{pmatrix}}_{\text{coeff. matrix } C} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

instead of F_L, can also measure scaling violations

$$F_D = -\frac{\beta_0}{2\beta(a_s(Q^2))} \frac{dF_2}{d\ln Q^2};$$

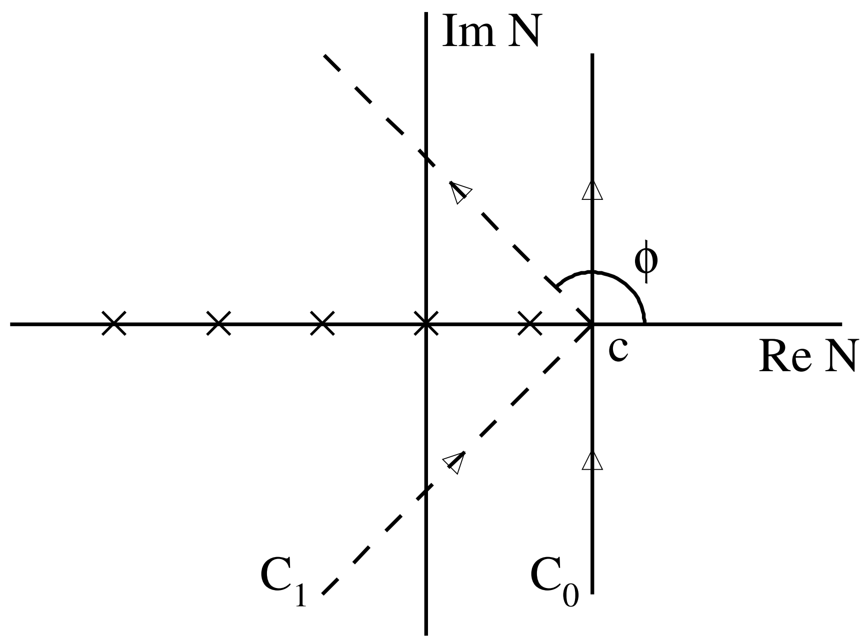
➡ use doublet (F_2, F_D) to substitute (Σ, g)

$$\begin{pmatrix} F_2 \\ F_D \end{pmatrix} = \underbrace{\begin{pmatrix} C_{2q} C_{2g} \\ C_{Dq} C_{Dg} \end{pmatrix}}_{\text{coeff. matrix } C} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

Realization: Work in conjugate Mellin space

In moment space, $a(N) = \int_0^1 dx\; x^{N-1} a(x)$, convolutions turn into products

$$F(x) = \iint_0^1 dz_1 dz_2\; C(z_1) f(z_2) \delta(z_1 z_2 - x) \qquad \Leftrightarrow \qquad F(N) = C(N) \cdot f(N)$$



- turns analysis into linear algebra
- inverse Mellin transform: numerically

$$a(x) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} x^{-N} a(N)$$

Physical evolution kernels - master formula

For a suitable doublet of observables determine (with $a_s = \frac{\alpha_s}{4\pi}$):

$$d_{\ln Q^2} \begin{pmatrix} F_A \\ F_B \end{pmatrix} = d_{\ln Q^2} \left[C \cdot \begin{pmatrix} \Sigma \\ g \end{pmatrix} \right]$$

$$= \left[\beta \frac{dC}{da_s} + C \cdot P \right] \cdot \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

$$= \left[\beta \frac{dC}{da_s} + C \cdot P \right] C^{-1} \begin{pmatrix} F_A \\ F_B \end{pmatrix} \equiv K \cdot \begin{pmatrix} F_A \\ F_B \end{pmatrix}$$

master formula

$$K = \left[\beta \frac{dC}{da_s} + C \cdot P \right] C^{-1} = a_s K^{(0)} + a_s^2 K^{(1)} + a_s^3 K^{(2)} + \ldots$$

- kernel K independent of factorization scheme & scale order by order in perturbation theory
  [Blümlein, Ravindran, van Neerven, NPB 586, 349 (2000)]
- finite order: dependence on renormalization scale & scheme remains → use for α_s determination

Some technical details ....

leading order coefficents of F_L vanish $\longrightarrow$ need to use rescaled version

$$\tilde{F}_L = \frac{F_L}{a_s C^{(1)}_{Lq}} \quad \text{or} \quad \tilde{F}_L = \frac{F_L}{a_s C^{(1)}_{Lg}}$$

possible as $C^{(1)}_{Lq,g}$ scheme independent

- final result independent of this choice
- kernels $K^{(0)}, K^{(1)}, K^{(2)}$ etc. from master formula in terms of coefficients and DGLAP splitting functions
- LO: physical evolution agrees with pdf evolution
- so far: work with fixed number of massless flavors

Numerical implementation

- initial condition: to test framework

  ➡ toy input at $Q_0^2 = 2\text{GeV}^2$,
  (PEGASUS [A. Vogt, CPC 170, 65 (2005)] default intial parton distributions)

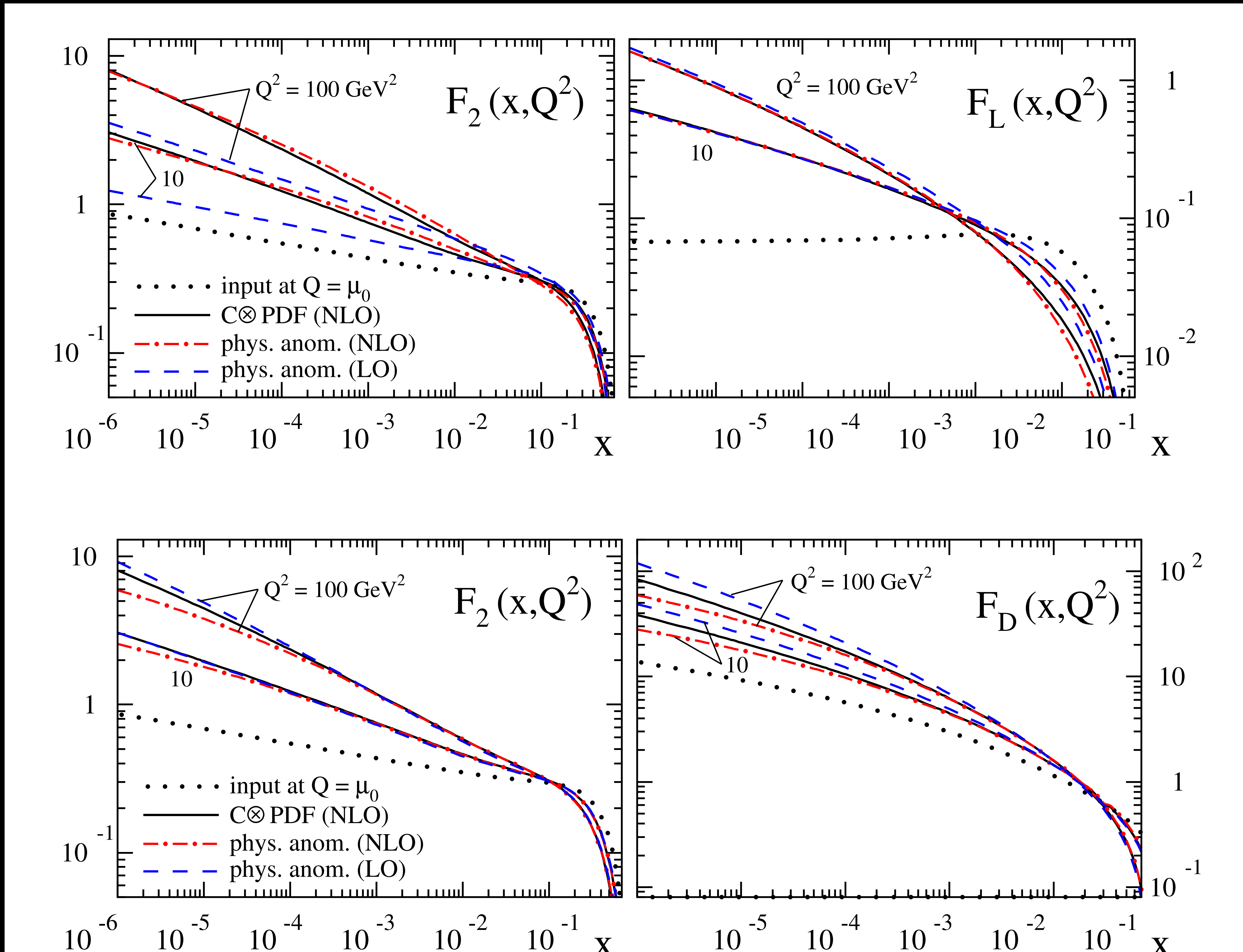
  fix $n_f = 3$ and $\alpha_s(Q_0^2) = 0.35$

$$
\begin{aligned}
xu_v(x, Q_0^2) &= 5.10722x^{0.8}(1-x)^3 \\
xd_v(x, Q_0^2) &= 3.064320x^{0.8}(1-x)^4 \\
xg(x, Q_0^2) &= 1.70000x^{-0.1}(1-x)^5 \\
x\bar{d}(x, Q_0^2) &= 0.1939875x^{-0.1}(1-x)^6 \\
x\bar{u}(x, Q_0^2) &= (1-x)x\bar{d}(x, Q_0^2) \\
xs(x, Q_0^2) &= x\bar{s}(x, Q_0^2) = 0.2(\bar{u}+\bar{d})(x, Q_0^2)
\end{aligned}
$$

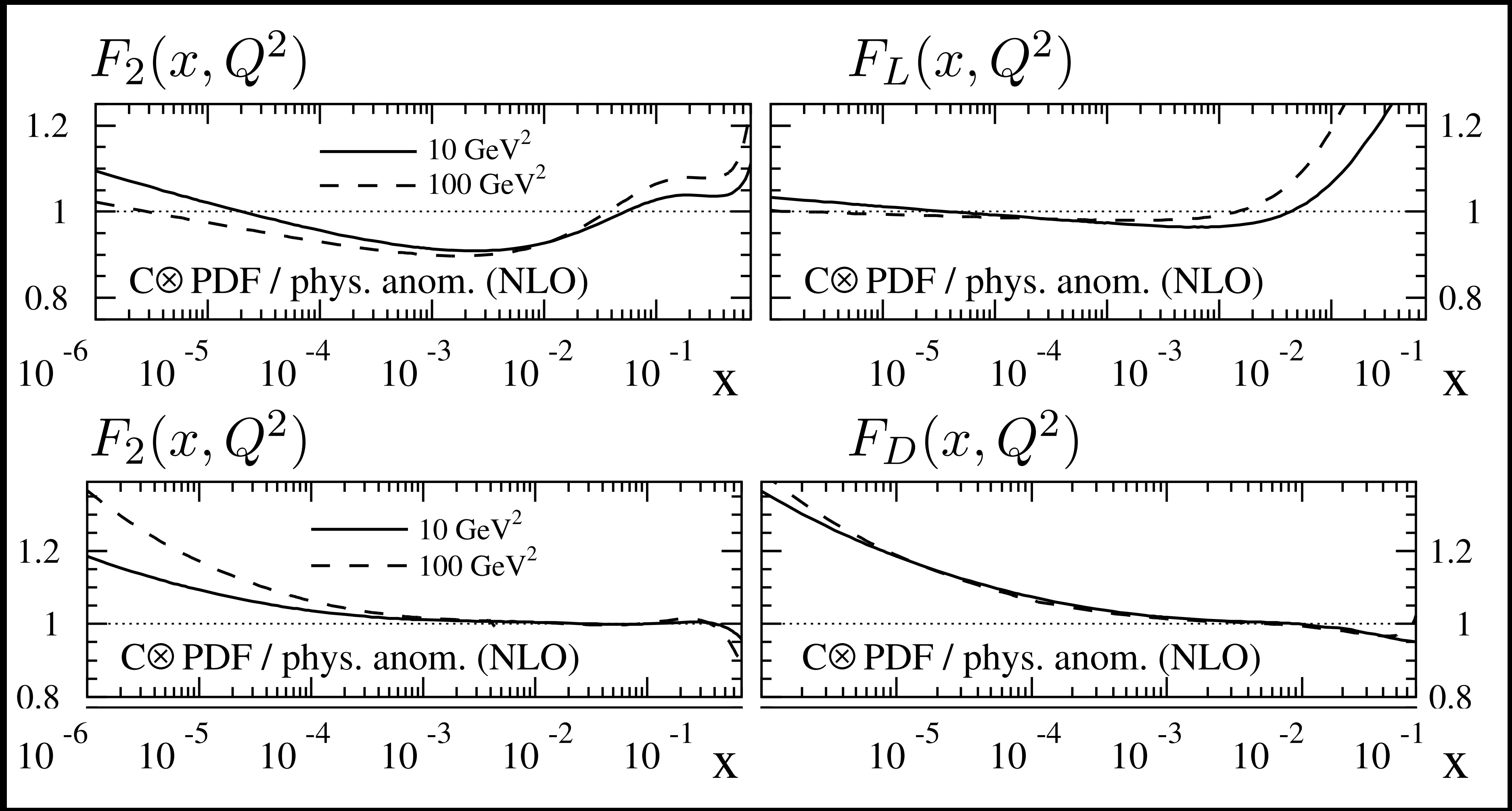
- structure functions at input scale from pdfs

$$
F_I(x, Q_0^2) = \sum_k C_{I,k}(Q_0^2) \cdot f_k(x, Q_0^2)
$$

results for (F_2, F_L) and (F_2, F_D) up to NLO

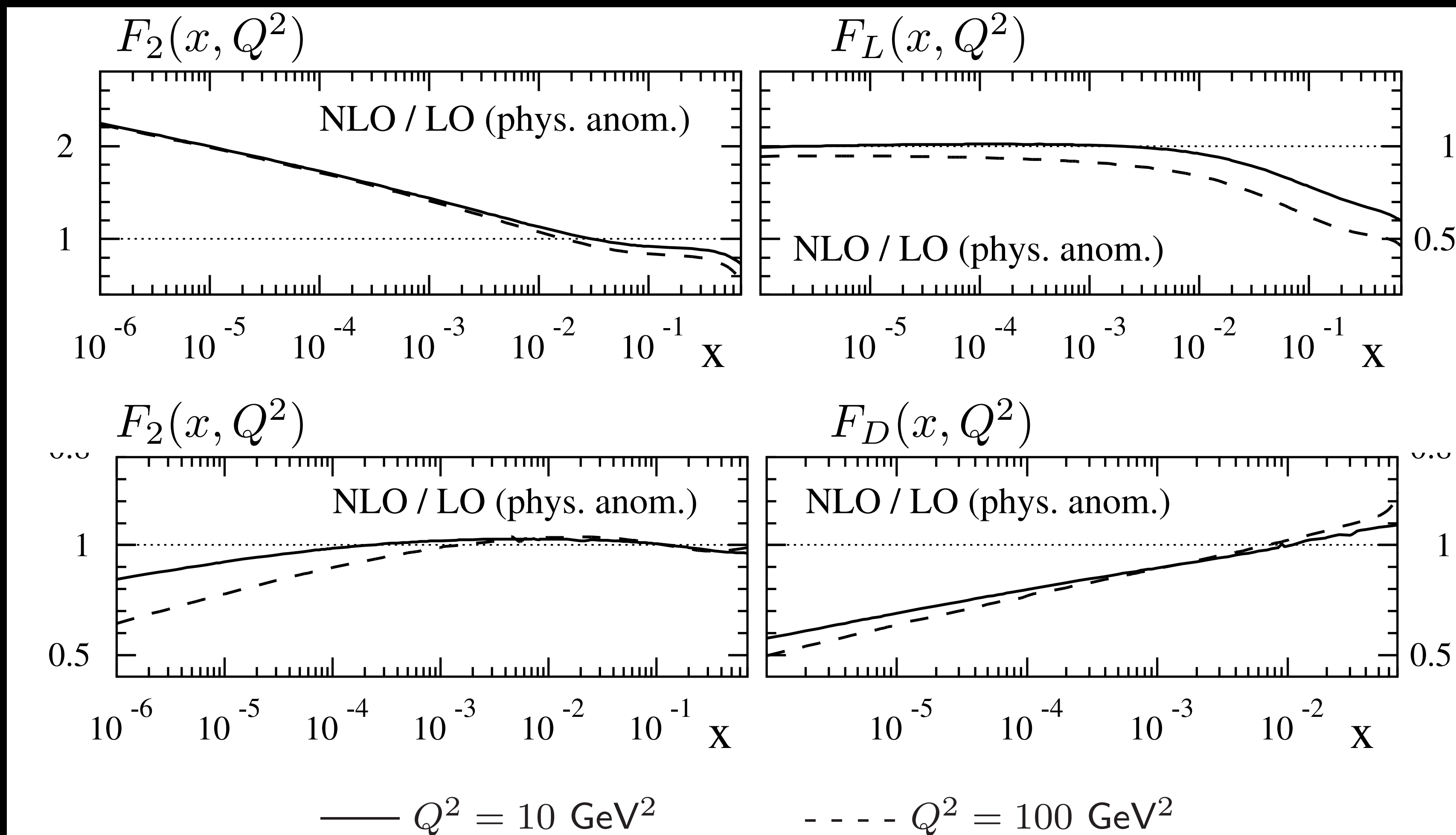


ratios phys. anom. dim. / conventional pdfs



- in some regions, large differences at NLO → large higher order spurious terms
- toy model: can be systematical excluded → exact agreement (higher order zero solution, original [Glück, Grassie, Reya, PRD 30, 1447 (1984)], [Glück, Reya, Vogt, PRD 46, (1992)])

ratios NLO/LO phys. anom. dim. evolution



- particularly large in the small x region
- convergence of the perturbative series?

Physical evolution at Next-to-Next-to-Leading Order

- differences to pdf first time at NLO ➡ NLO effective LO
- NLO corrections large - convergence of the perturbative series?
- want to please referee .....

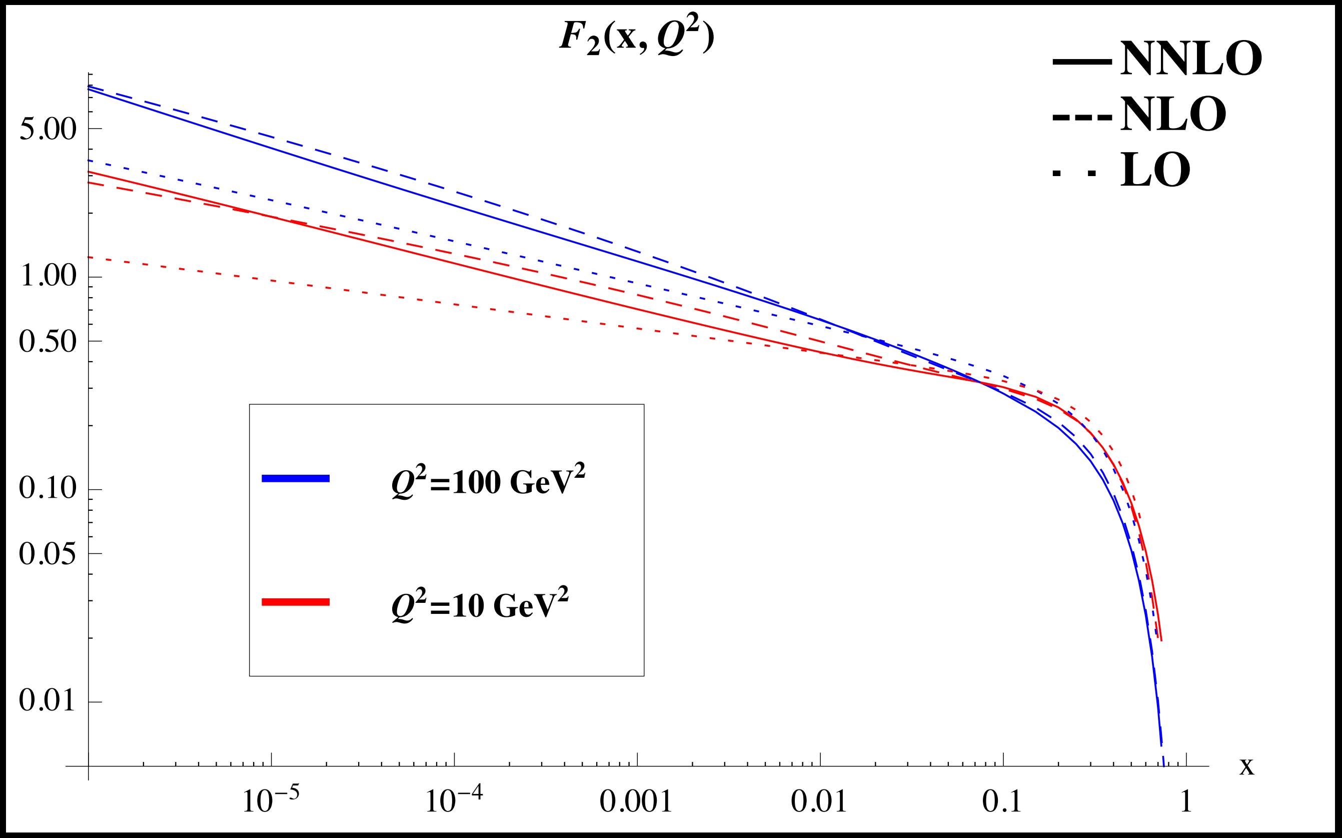
literature:

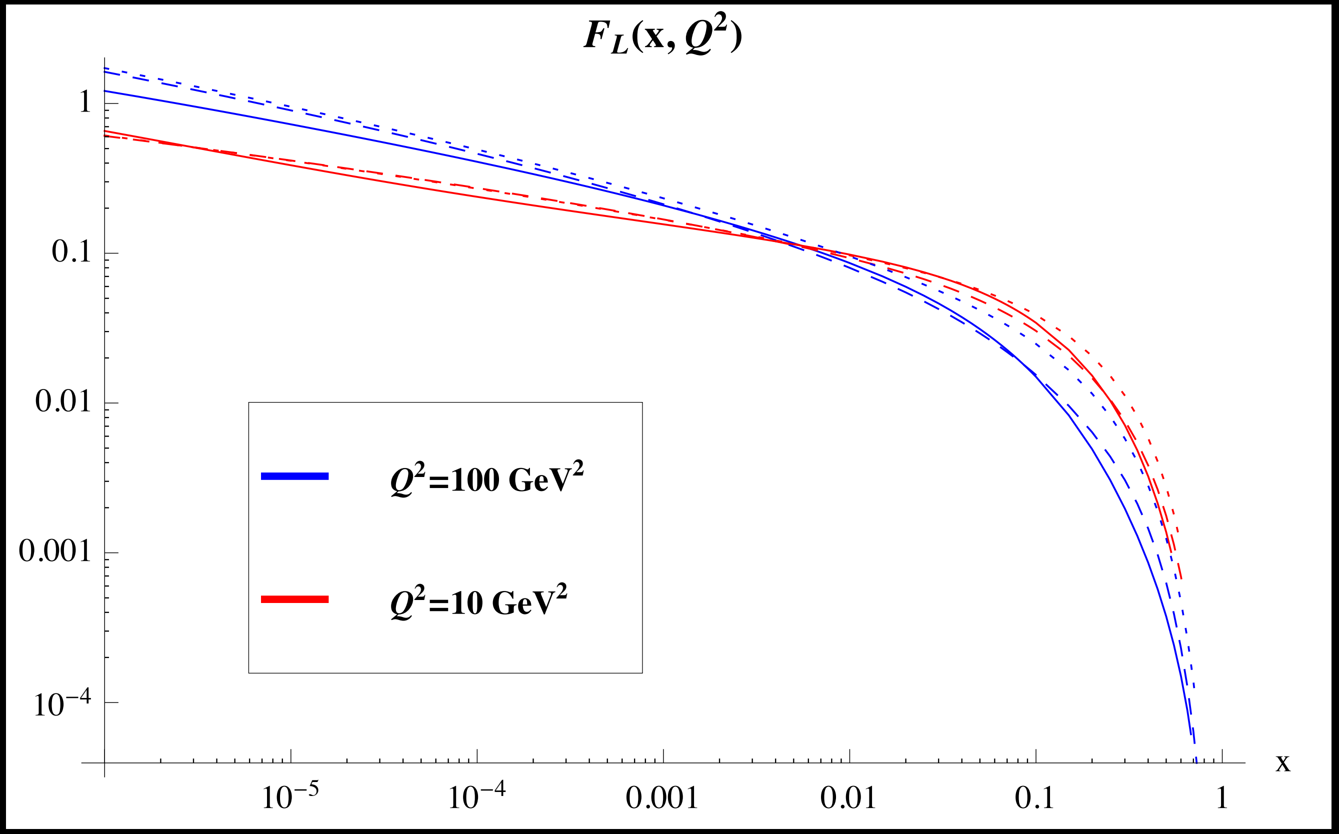
- 3-loop splitting functions for F_2 [Moch, Vermaseren, Vogt, NPB 688 (2004), NPB 691 (2004)]
- 2-loop coefficent for F_2 [van Neerven, Vogt, NPB 568 (2000); NPB 588 (2000)]
- 3-loop coefficent for F_L [Moch, Vermaseren, Vogt, Phys.Lett. B606 (2005) 123-129]

results very lengthy: use x-space parametrization & transform to moment (N)-space using Mathematica + cross checked with Pegasus

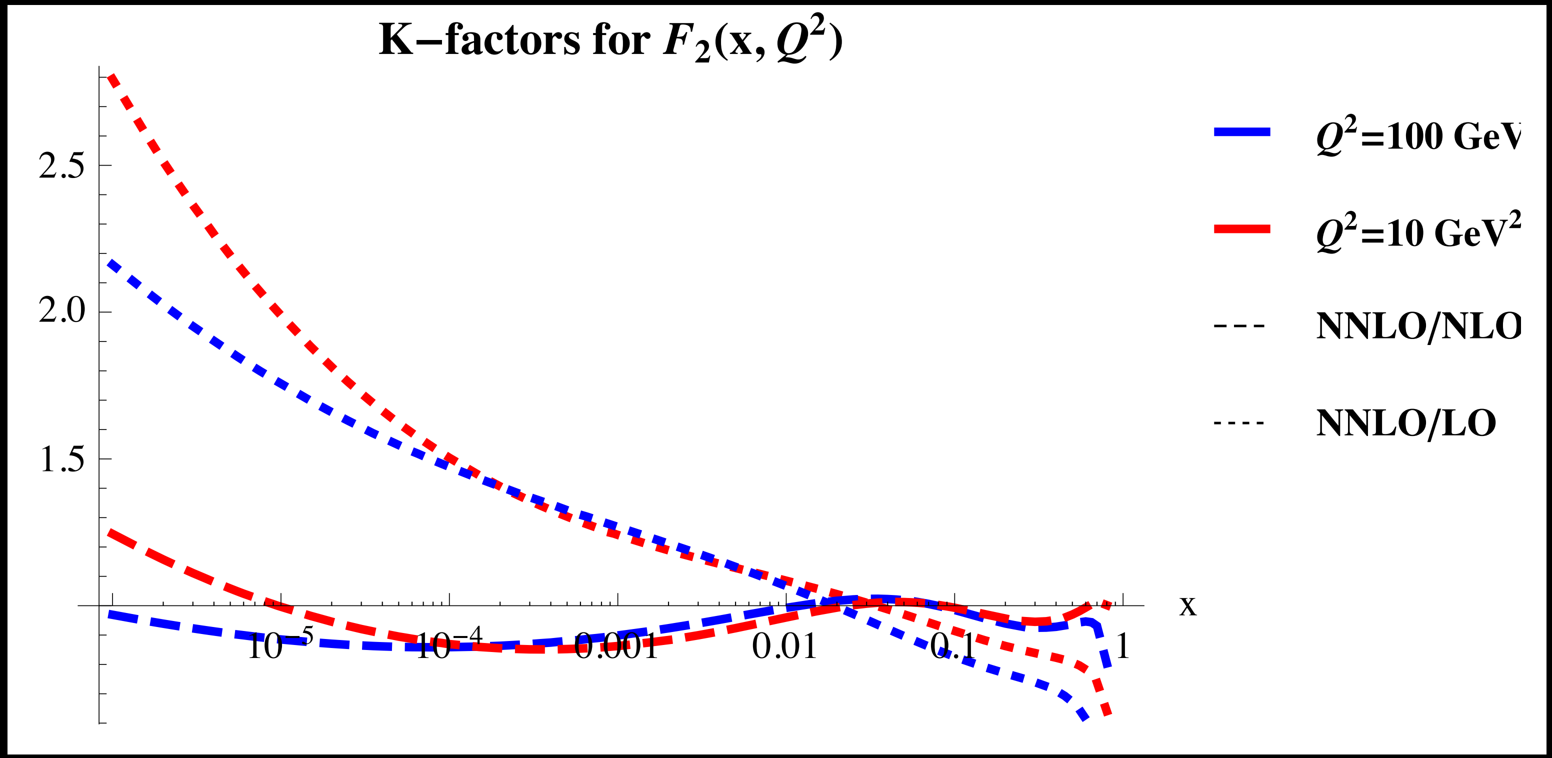
as for NLO: pdf & pad agree for toy model, if spurious higher order terms are excluded

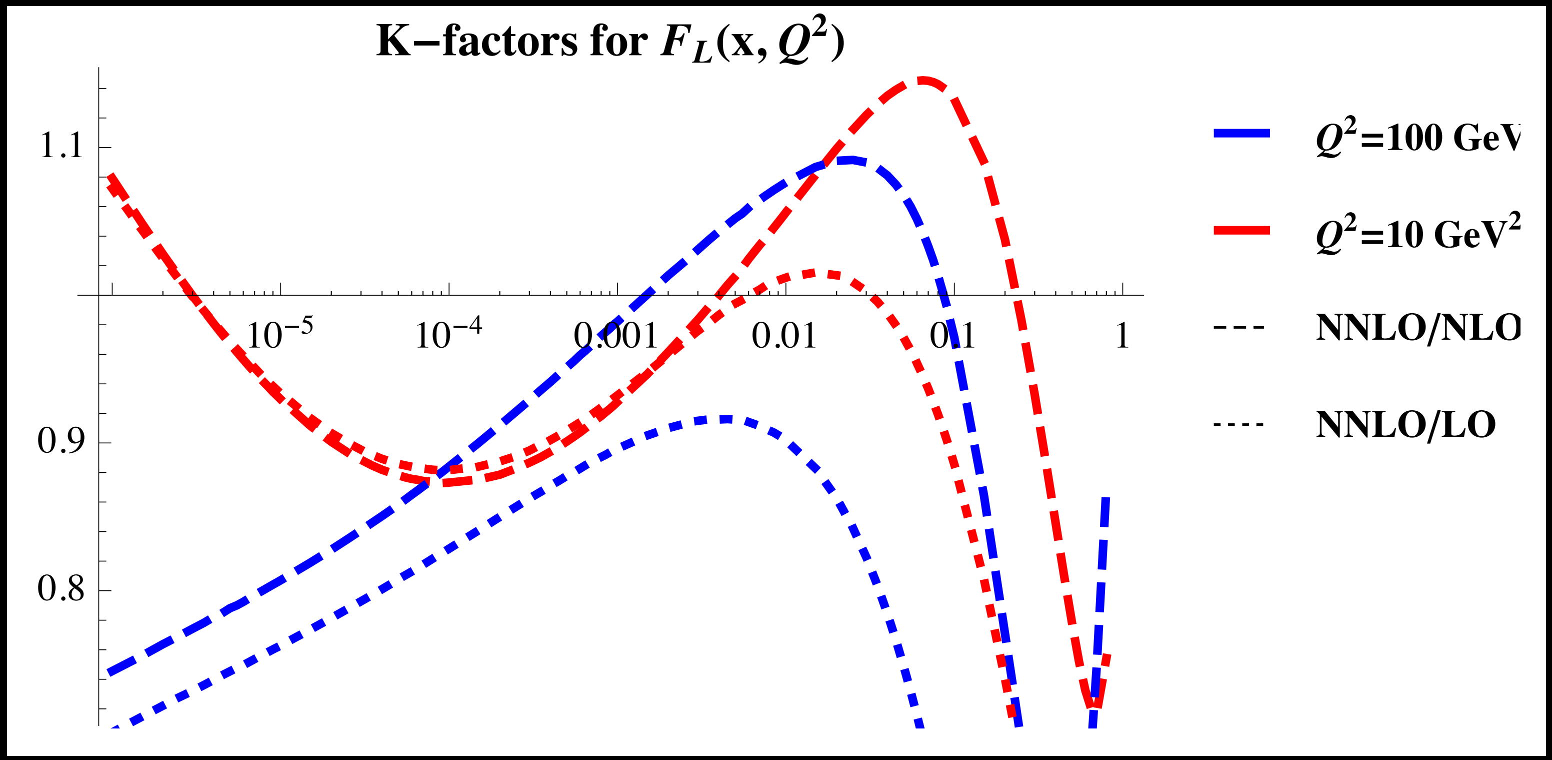
NNLO results - (F_2, F_L)





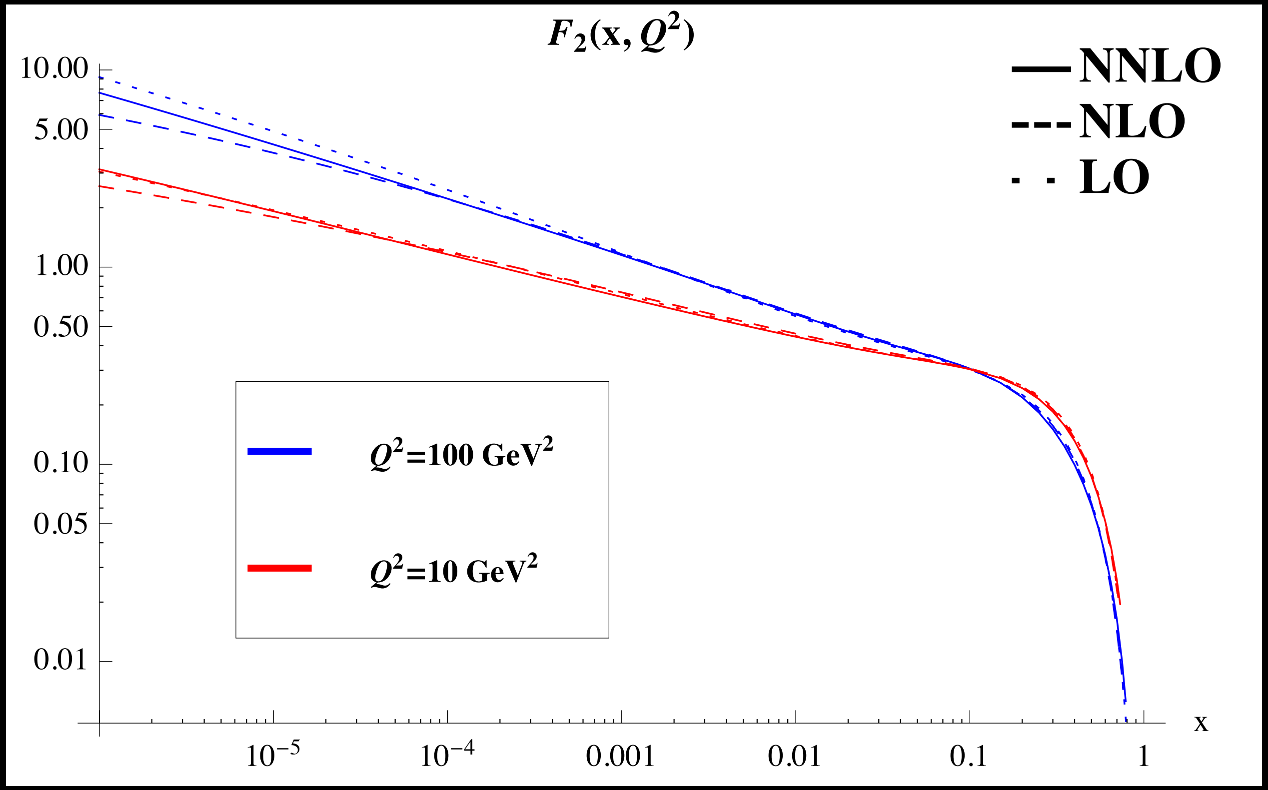
K-factors for (F_2, F_L)

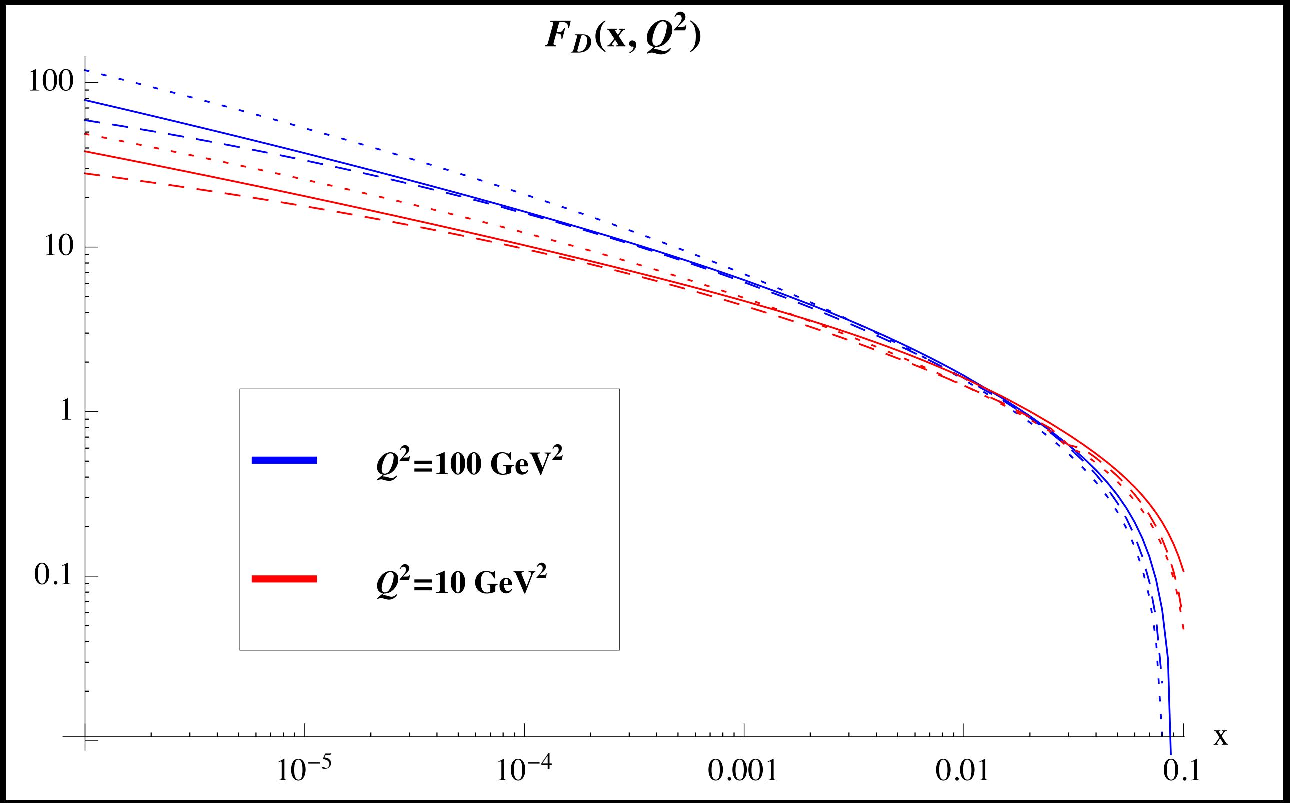




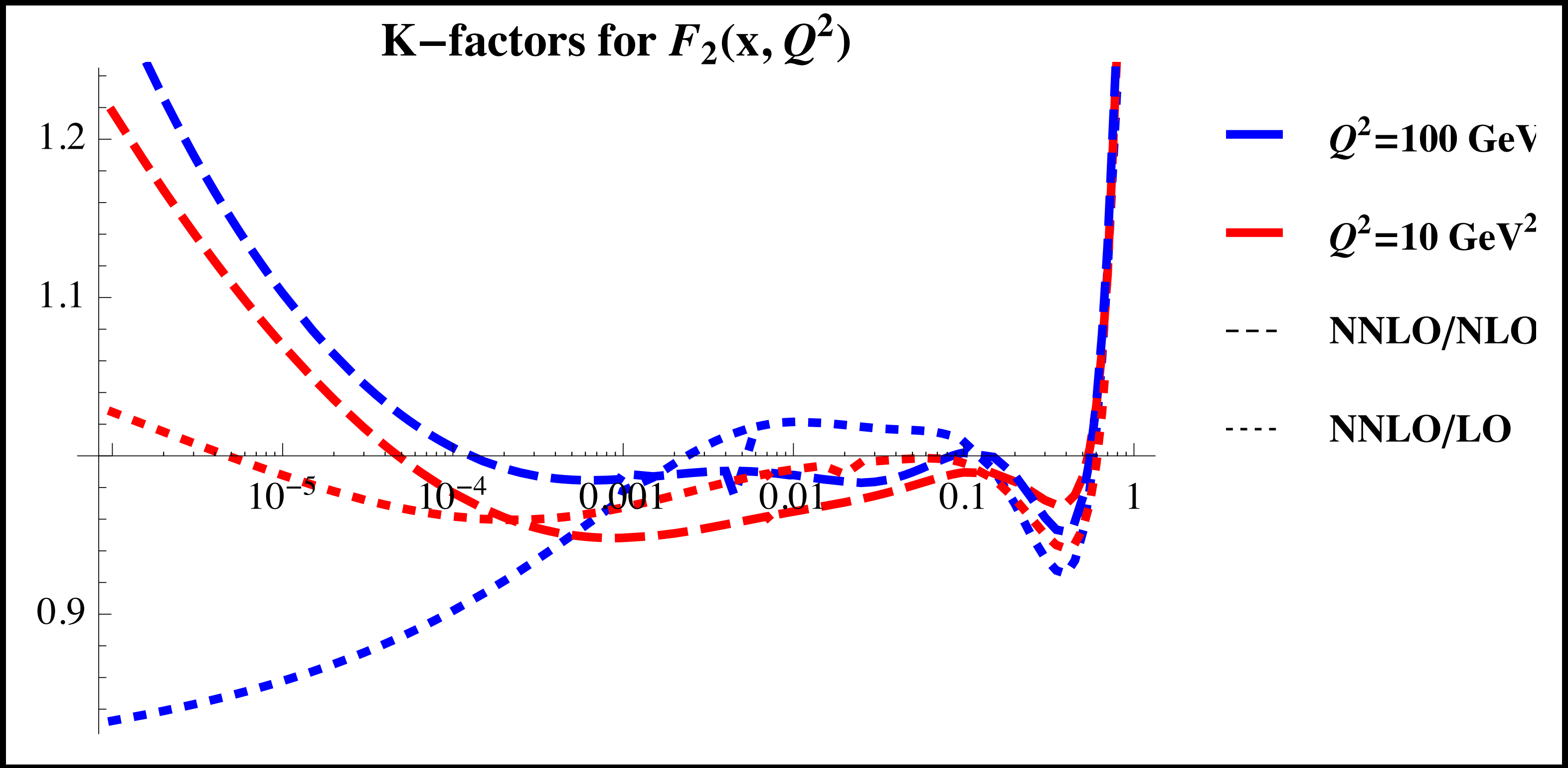
still a large correction, particular for F_L, but signs of convergence

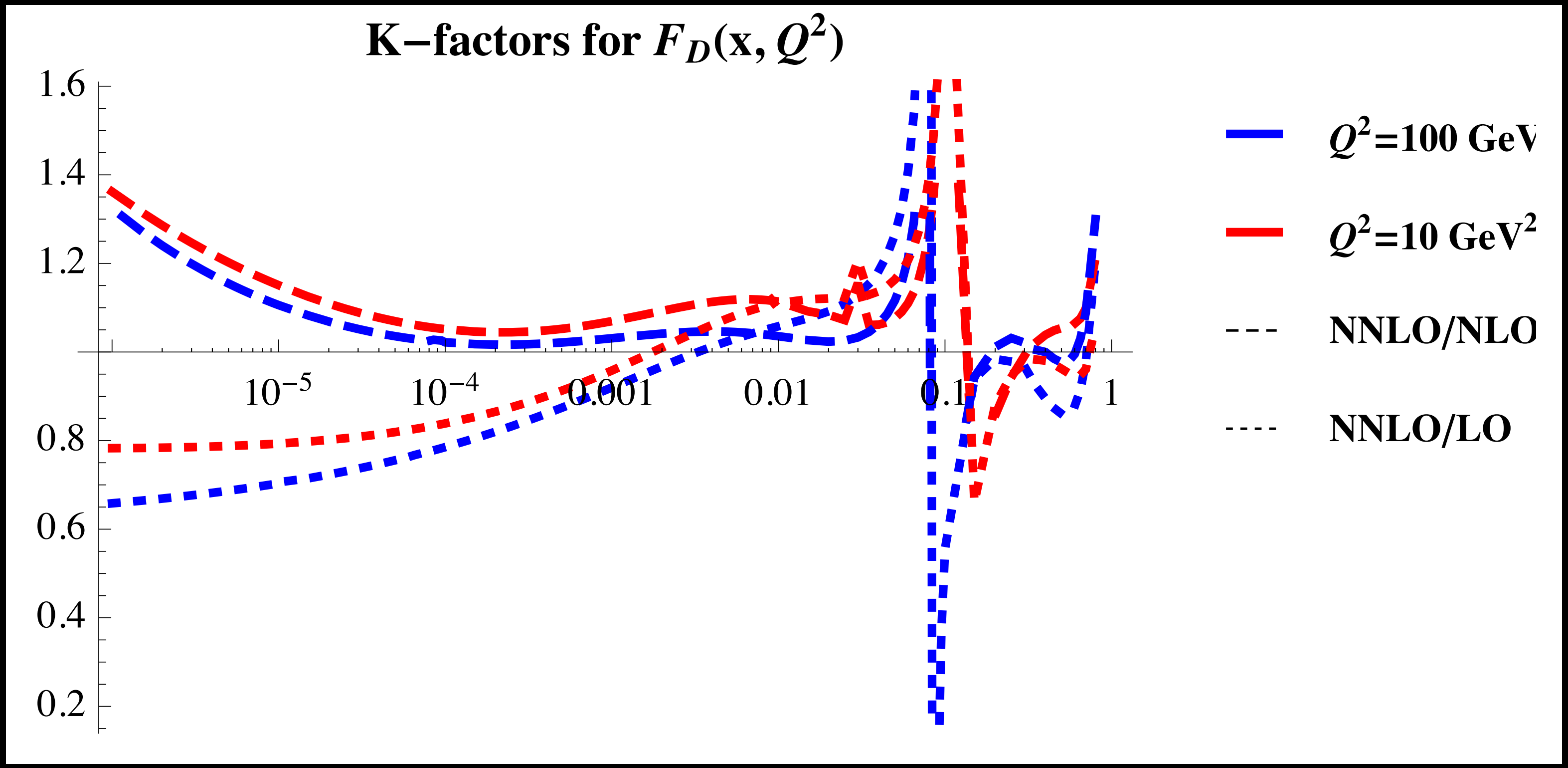
NNLO results - (F_2, F_D)





K-factors for (F_2, F_D)





NNLO still a large correction at small x, but signs of convergence

First applications to phenomenology

Two possible approaches:

- “physical” fit of available data (similar to pdfs)
  ➡ $\alpha_s(M^2)$ determination from inclusive DIS

- evolve structure functions from initial scale Q_0^2 = 1-2 GeV^2 & compare with data ➡ detect possible deviations from DGLAP

  in general: many independent observables to invert pdfs

  dominance of gluon at small x:
  flavor singlet good approximation to full structure functions

Search for signals of saturation in inclusive DIS

idea:

- use saturation models/evolution with initial conditions fitted to HERA data & calculate (F_2,F_L) at initial scale $Q^2 = 2$ GeV^2
- evolve with physical anomalous dimensions & compare to saturation prediction at higher Q^2
- deviation: sign for saturation if confirmed by data ...
- caveat: DGALP massless, sat. fits massive charm etc.

Simulate saturation: dipole models

$$\sigma_{L,T}^{\gamma^* p}(Q^2, x) = 2 \sum_f \int \int d^2b \, d^2r \int_0^1 dz \, |\Psi_{L,T}^{(f)}(r, z; Q^2)|^2 \, \mathcal{N}(x, r, b) \, ,$$

general problem:

• saturation models, (rc)BK at $x < 10^{-2}$ → DGLAP/PAD need full x -range for initial conditions

• solution: extrapolate color dipole models into large x region (can be problematic ....)

•rcBK: also possible, but more cumbersome

IP-sat model

- GBW dipole model [J. Golec-Biernat & Wusthoff, Phys.Rev. D59 (1998) 014017] with DGLAP evolution [J. Bartels, K. J. Golec-Biernat and H. Kowalski, Phys. Rev. D66, 014001 (2002)] (with impact parameter dependence: IP sat: [H. Kowalski and D. Teaney, Phys. Rev. D68, 114005 (2003)])

$$\mathcal{N}(x,r,b) = \left(1-\exp\left(-\frac{\pi^2 r^2}{2N_c}\alpha_s\left(\mu^2\right) xg\left(x,\mu^2\right) T_G(b)\right)\right)$$

- recent fit to combined HERA data [A. H. Rezaeian, M. Siddikov, M. Van de Klundert and R. Venugopalan, Phys. Rev. D 87, no. 3, 034002 (2013)]

(b)CGC-dipole model [Iancu, Itakura,Munier, PLB 590 (2004)]

$$N(x,r,b) = \begin{cases} N_0 \left(\frac{rQ_s}{2}\right)^{2\gamma_{eff}} & rQ_s \leq 2\,, \\ 1 - \exp\left(-\mathcal{A}\ln^2\left(\mathcal{B}rQ_s\right)\right) & rQ_s > 2\,, \end{cases}$$

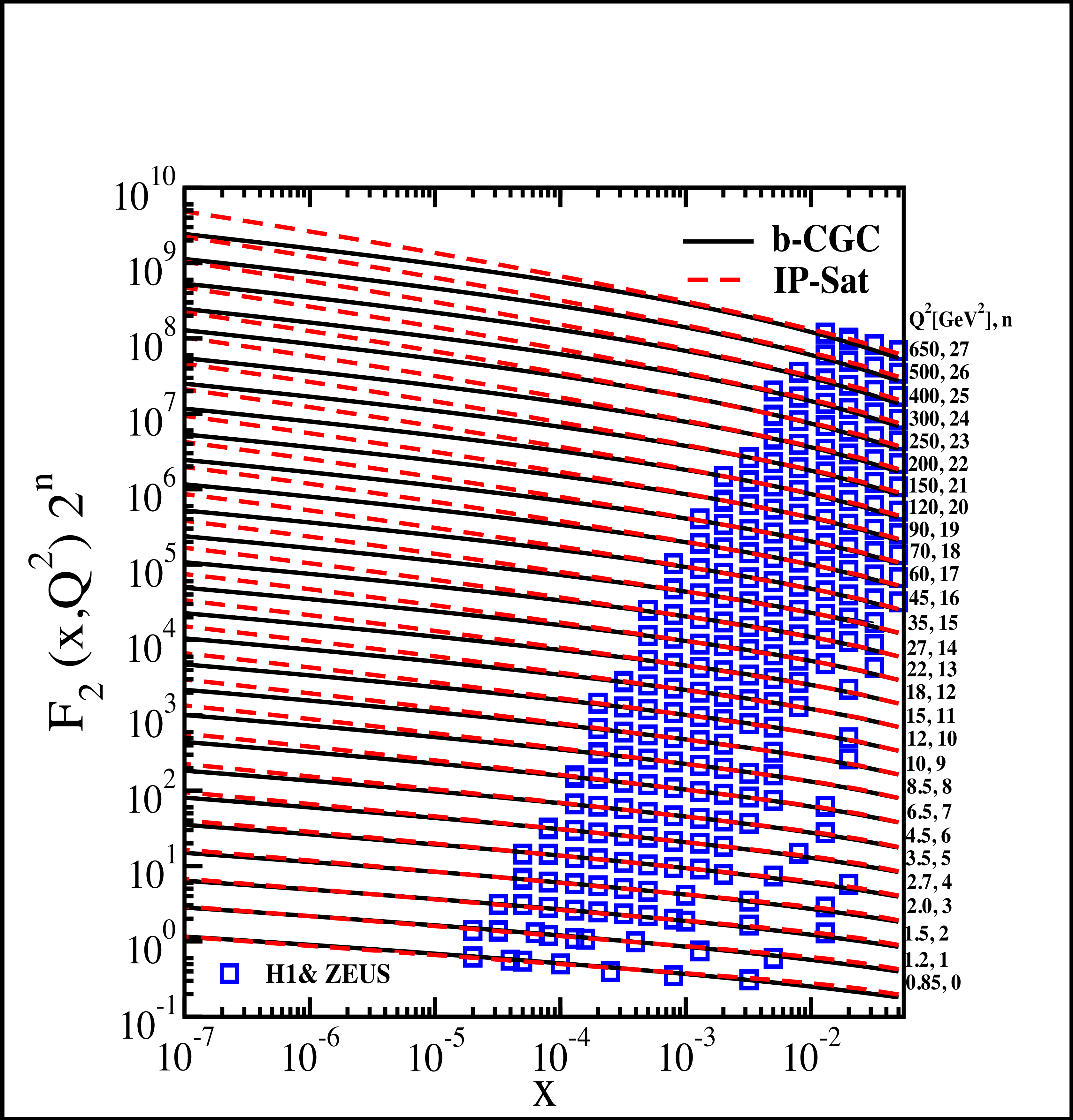
$$Q_s(x) = \left(\frac{x_0}{x}\right)^{\frac{\lambda}{2}} \mathrm{GeV} \qquad \gamma_{eff} = \gamma_s + \frac{1}{\kappa\lambda Y}\ln\left(\frac{2}{rQ_s}\right)$$

- interpolates between asymptotic solution to BK equation & BFKL solution for small dipoles close to saturation line
- b-dependence [Watt, Kowalski, PRD 78 (2008)], [Kowalski, Motyka, Watt, PRD 74 (2006)]
- recent fit to combined HERA data [Rezaeian, Schmidt, PRD 88 (2013)]

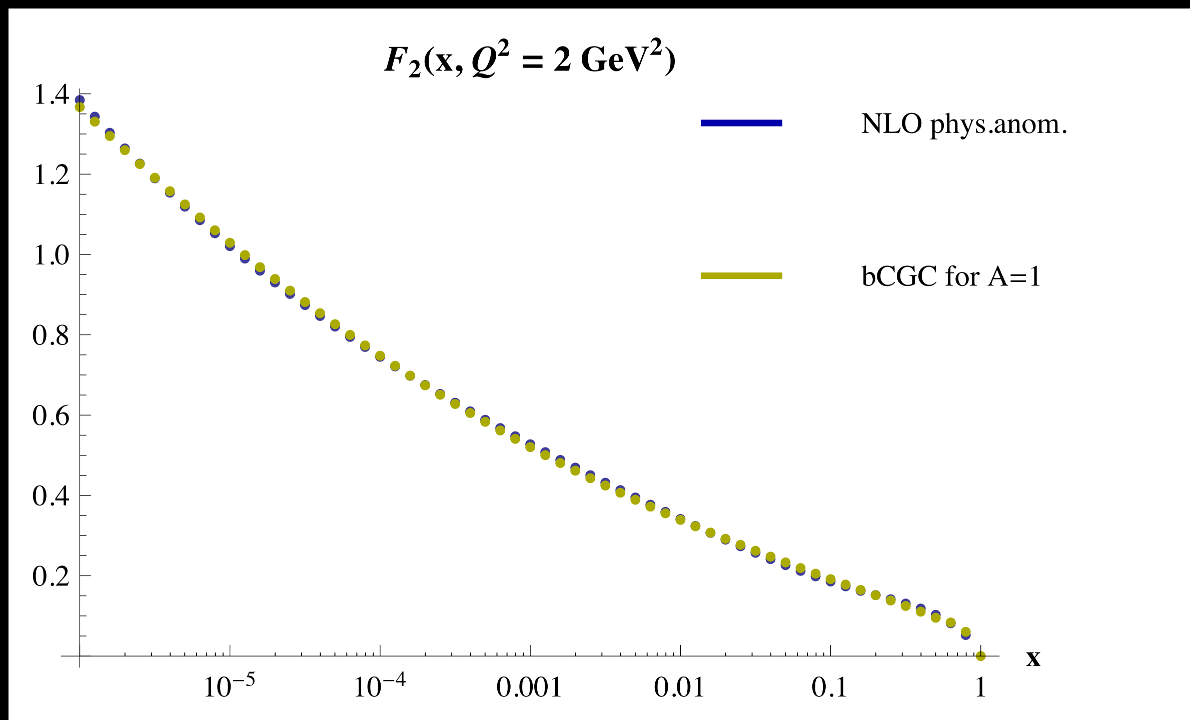
their fit to HERA data ....

differences mainly in ultra-small x region

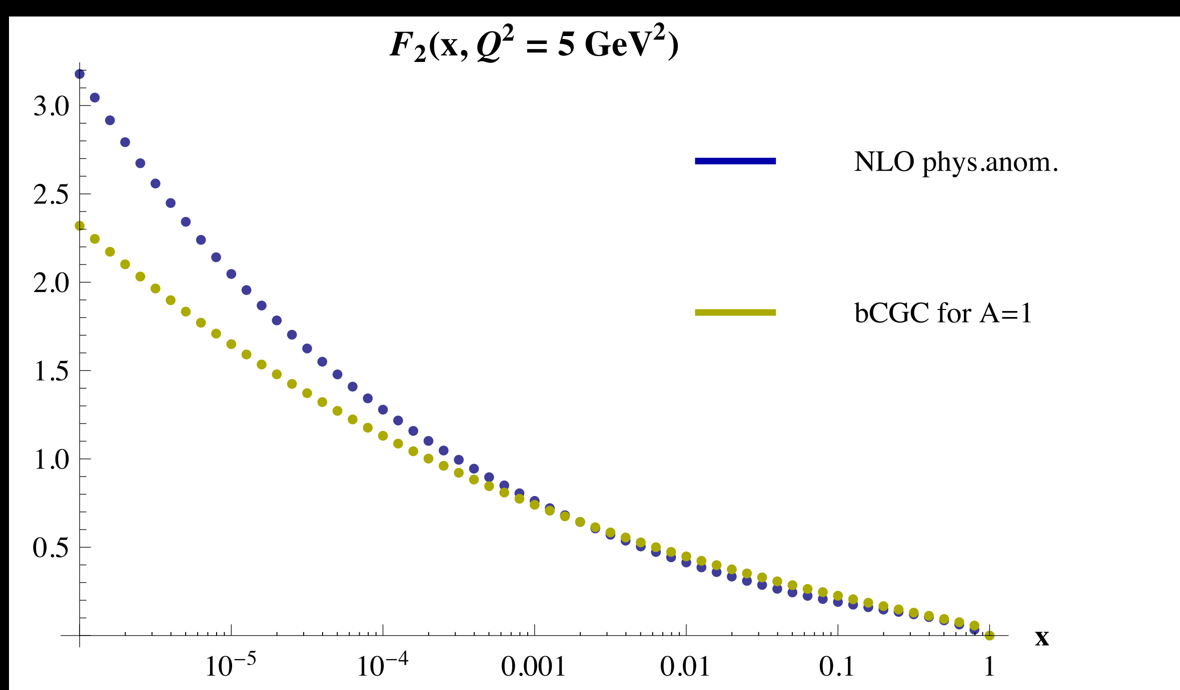
high Q^2 behavior fixed by data

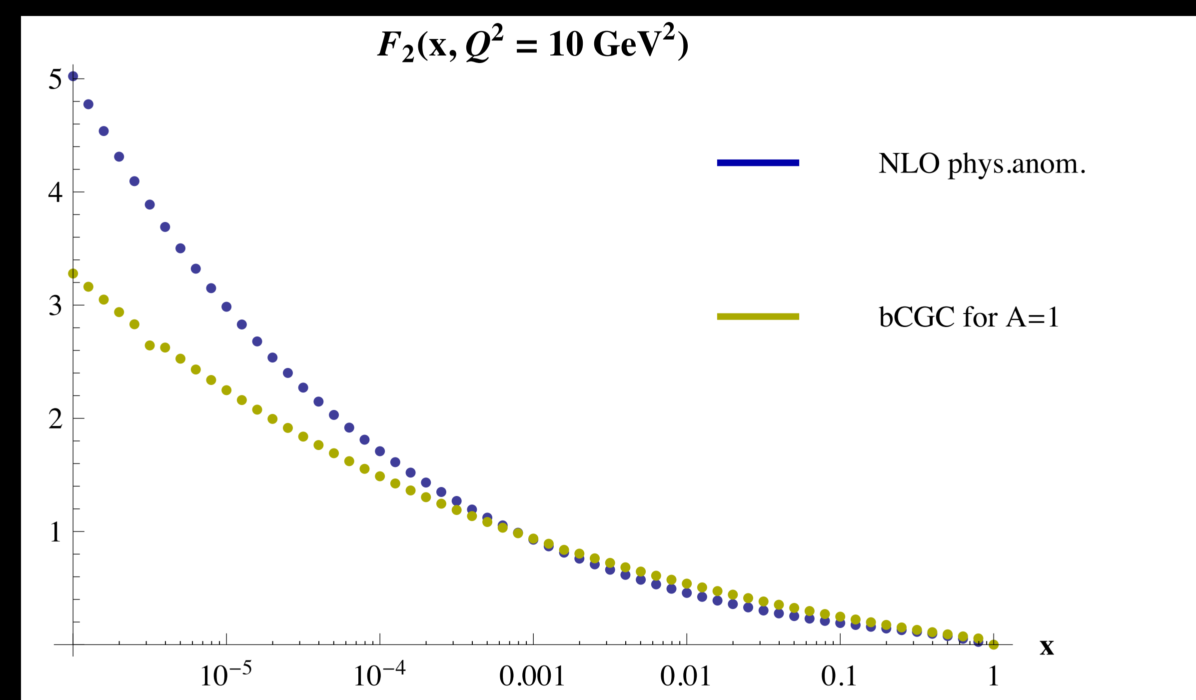


CGC-dipole model: DIS on proton

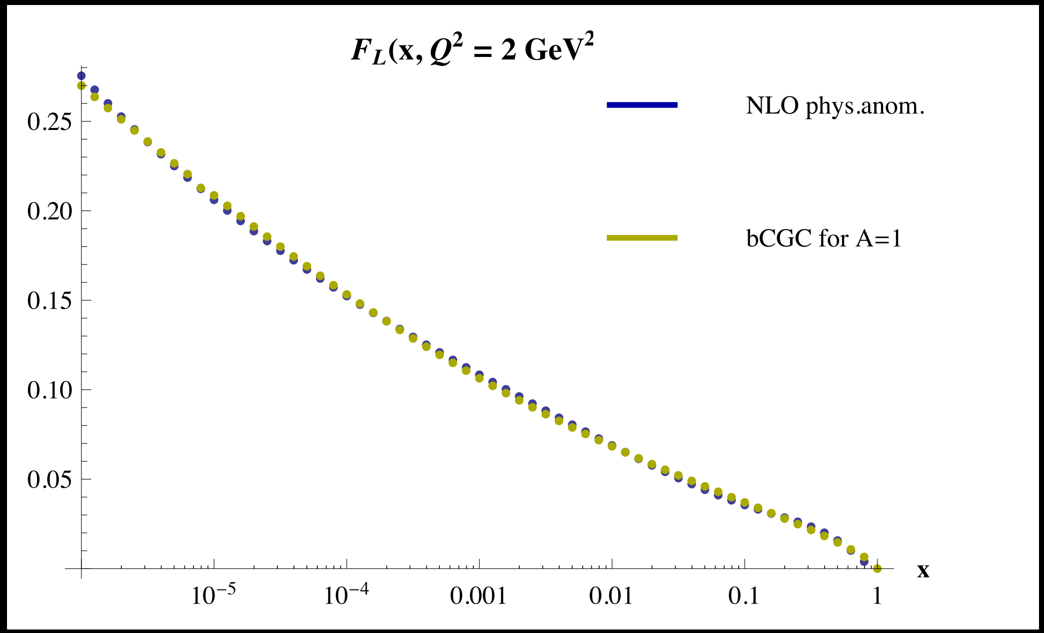


- fit (F_2 , F_L) at Q^2=2 GeV^2
- evolve doublet with physical anomalous dimensions & compare

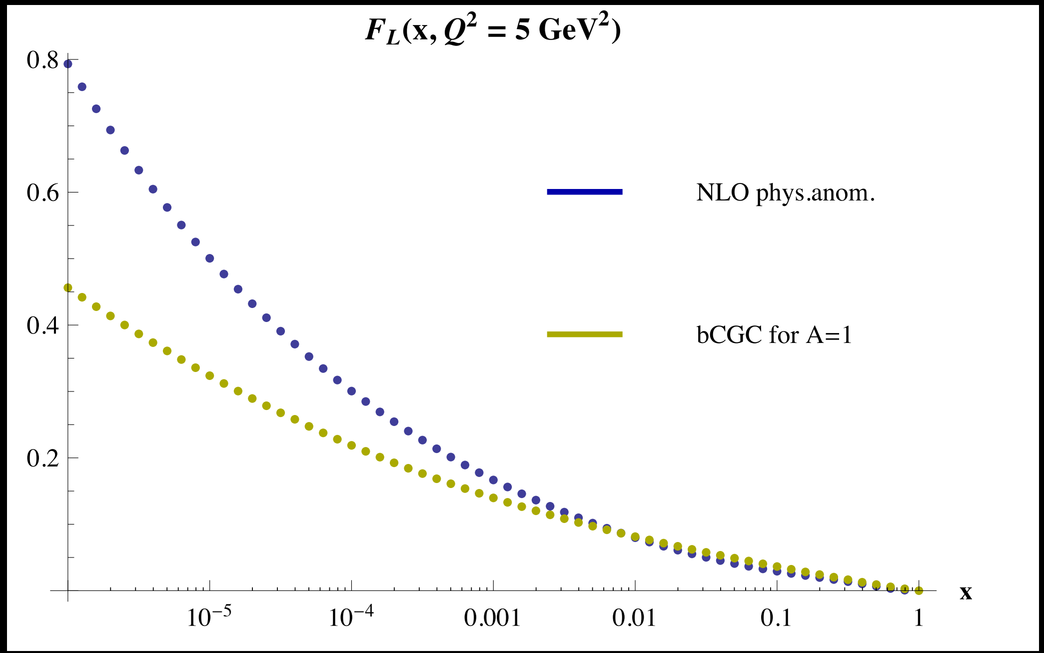


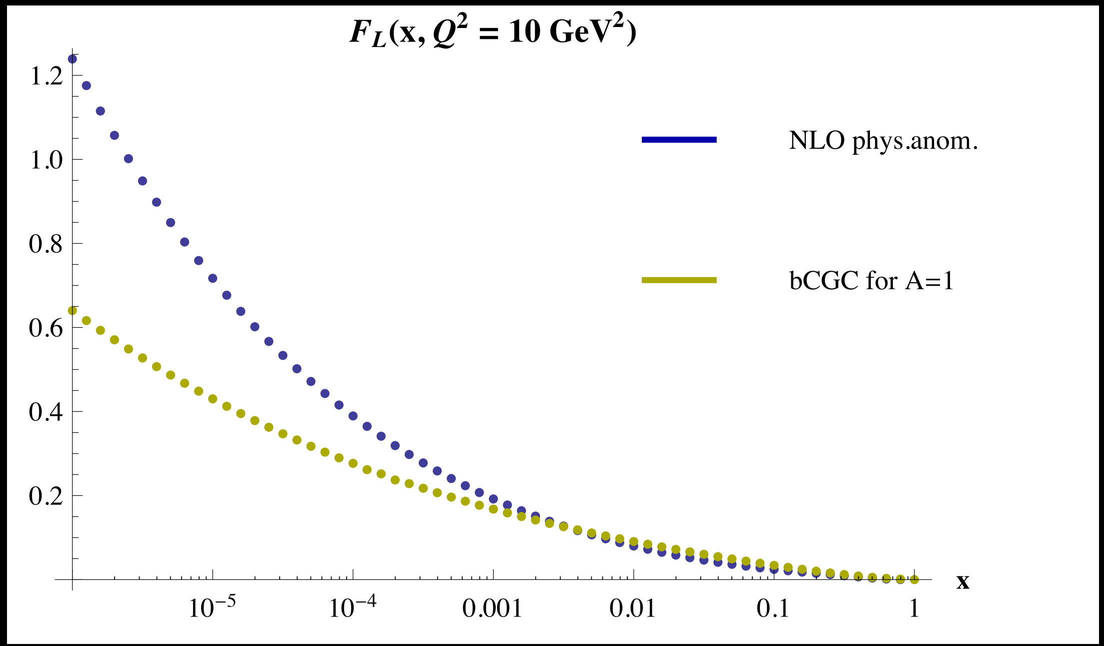


The proton structure function F_L

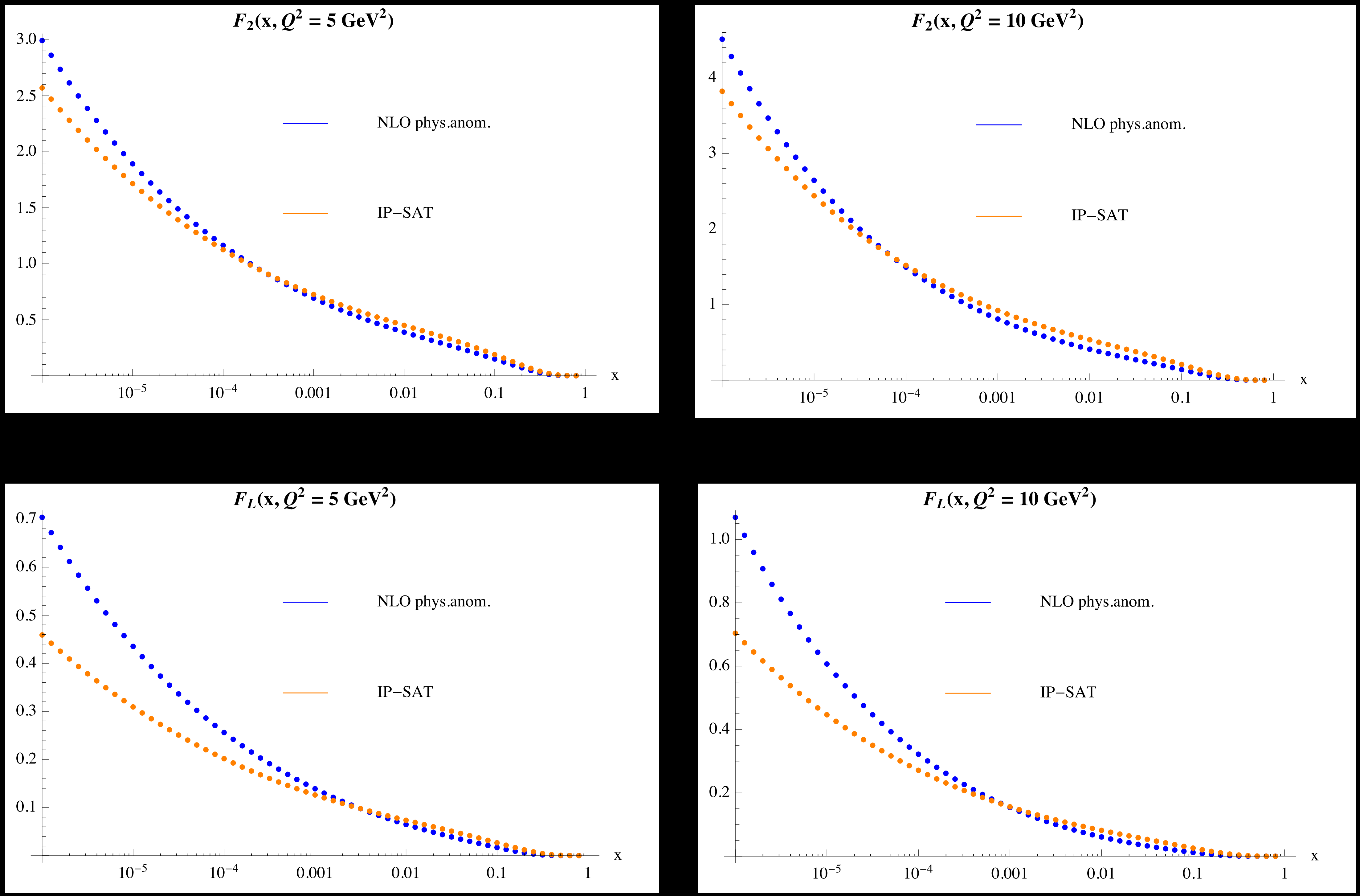


- effects stronger, even for $x > 10^{-4}$
- dipole fit less constrained by data

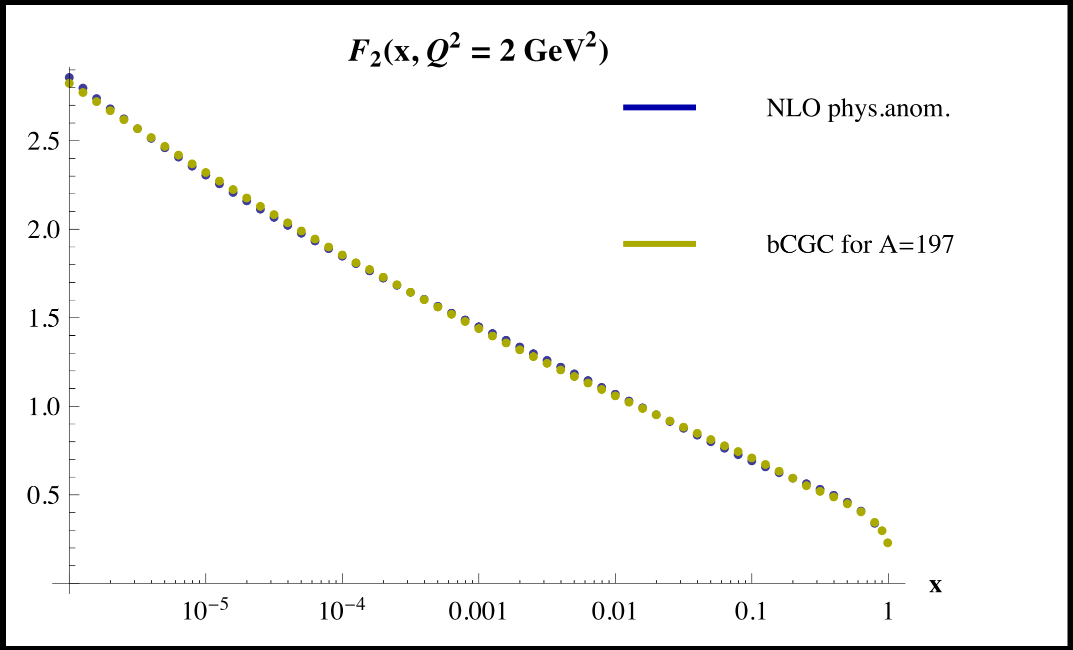




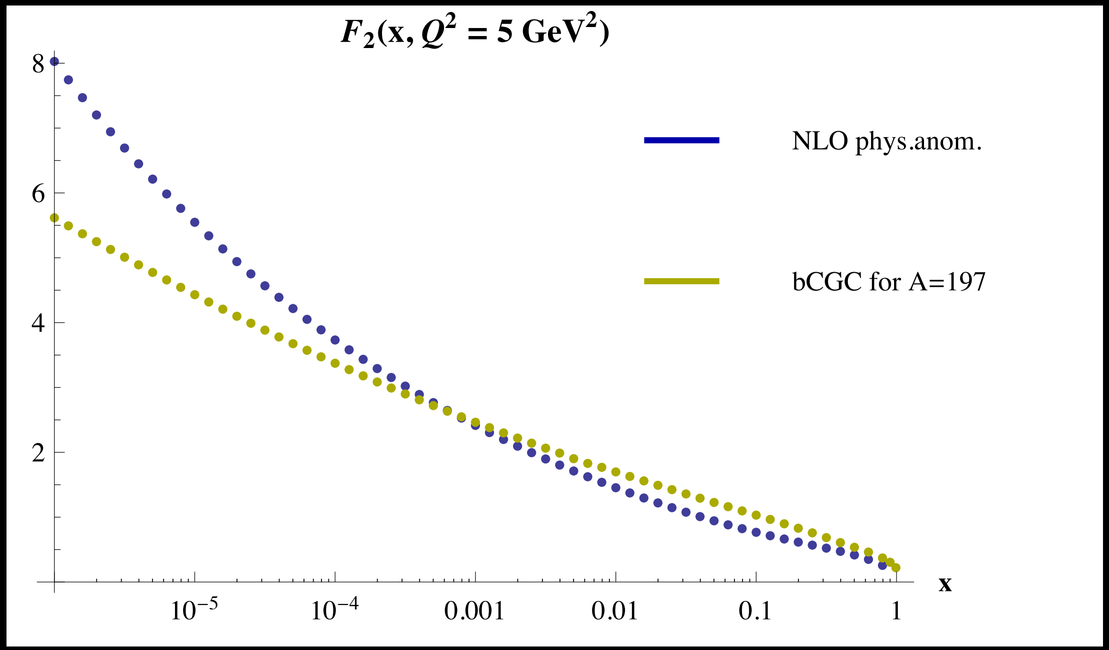
The same for the IPSat fit

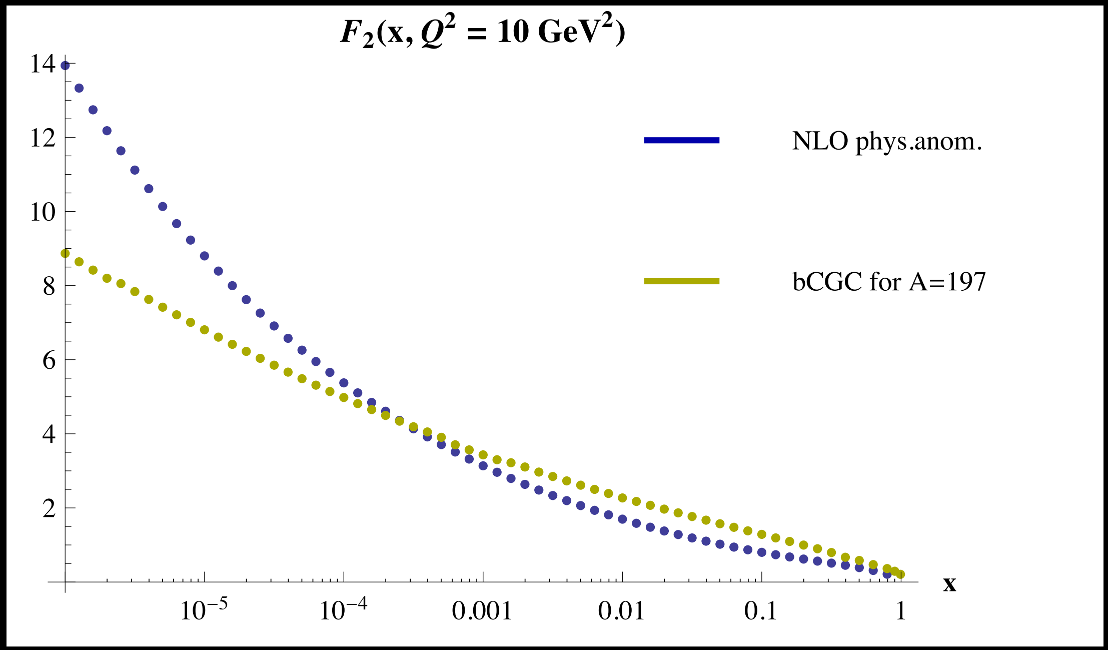


bCGC: Nuclear effects through $Q_s^2 \longrightarrow A^{1/3}Q_s^2$



- initial conditions converge slowly to zero at x=1
- relative difference not as big as naively expected





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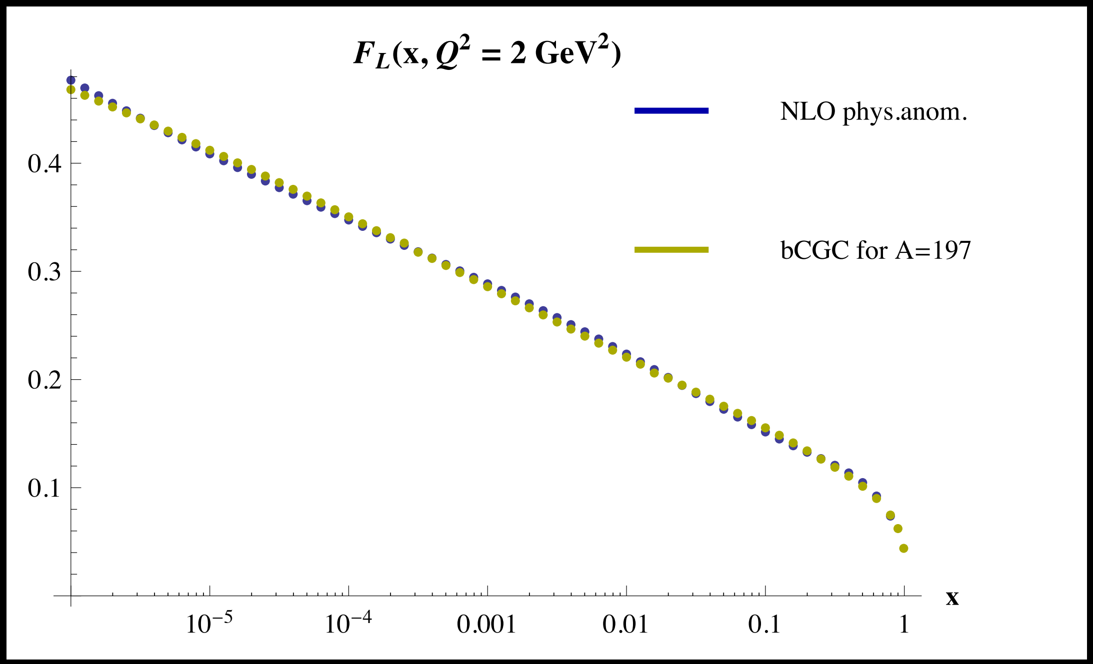
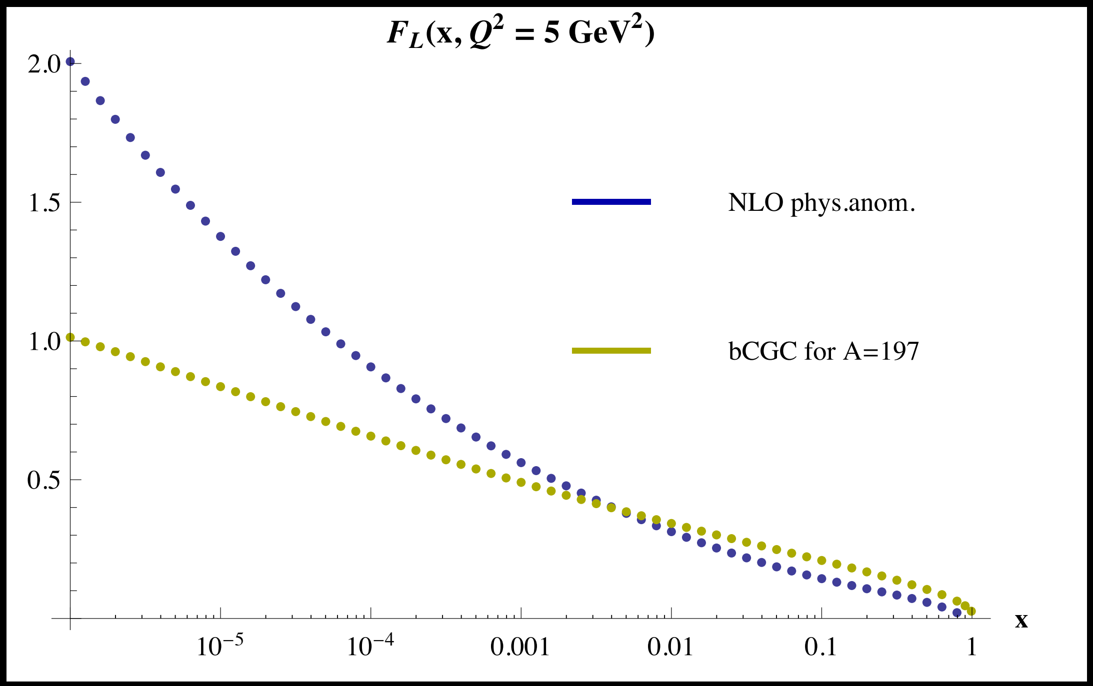
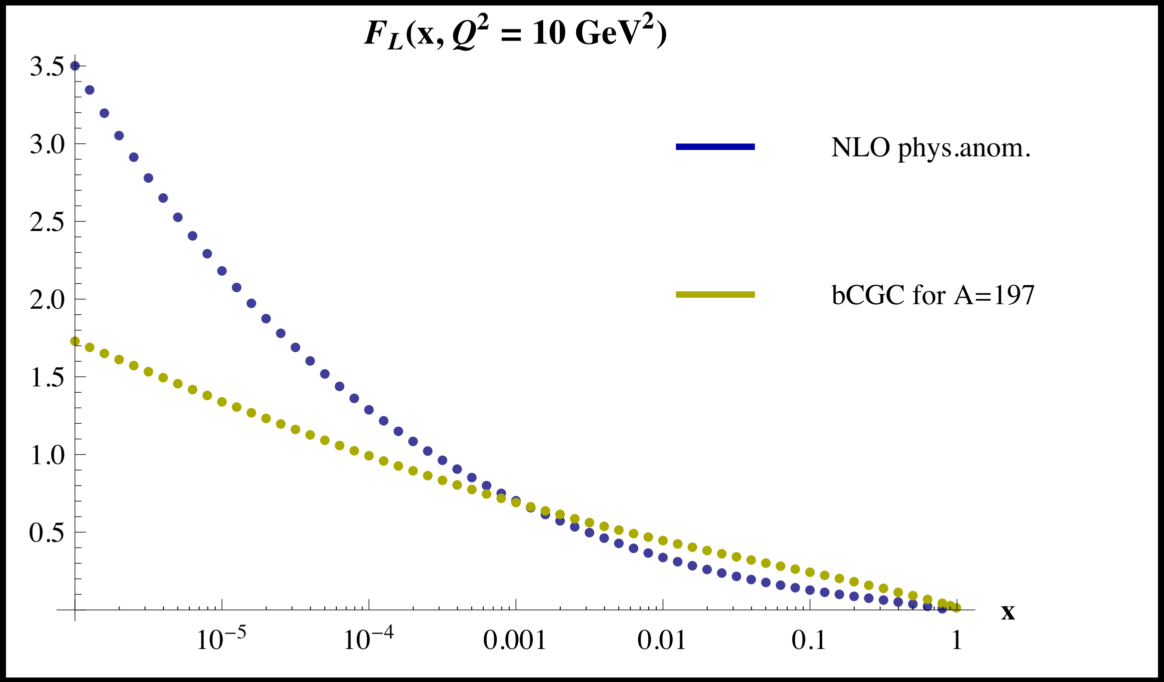


- differences again more significant for F_L
- relative difference again close to proton case





Summary & Outlook

- Method works & can in principal be applied to phenomenology
- To be done: heavy flavors
- Application:
  - ★quantify size of non-linear effects through deviations from physical evolution
  - ★First example: saturation models
  - ★Running coupling